Towards the Born-Weyl Quantization of Fields*

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Elements of the quantization in field theory based on the covariant polymomentum Hamiltonian formalism (the De Donder-Weyl theory), a possibility of which was originally discussed in 1934 by Born and Weyl, are developed. The approach is based on a recently proposed graded Poisson bracket on differential forms in field theory. A covariant analogue of the Schrödinger equation for a hypercomplex wave function is put forward. It leads to the De Donder-Weyl Hamilton-Jacobi equations in quasiclassical limit. A possible relation to the functional Schrödinger picture in quantum field theory is outlined.

1 INTRODUCTION

The approach to the canonical quantization of field theories which was originally developed shortly after the formalism of quantum mechanics was established is based on the representation of fields as mechanical systems with an infinite number of degrees of freedom. This approach was essentially inspired by what Heisenberg and Pauli referred to in their private correspondence as "Volterra Mathematik" namely, some developments of that time in the functional and variational calculus (cf. e.g. (Volterra, 1959)). The canonical quantization of fields commonly known since then

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is based on the only generalization of the Hamiltonian formalism to field theory available at that time. However, shortly thereafter in the papers by Carathéodory, De Donder, Weyl and others (see e.g. (Rund, 1966) for a review) on the calculus of variations of multiple integrals different alternative ways of extending the Hamiltonian formulation to field theory have appeared which where unified within a general scheme later in the forties by Lepage (see e.g. (Kastrup, 1983) for a review and references). Unlike the standard Hamiltonian formalism in field theory all these formulations do not distinguish between the space and time coordinates and do not refer to an infinite dimensional phase space. Instead, fields are treated rather as a sort of dynamical systems with several "times" the role of which is played by all space-time coordinates on equal footing. In doing so the phase space is replaced by what is called below the *polymomentum phase space*, a finite dimensional space of field variables and "polymomenta" which are defined from the Lagrangian as the conjugate momenta associated with each space-time derivative of the field (see Sect. 2). In the case of one-dimensional "space-time" this picture reproduces the standard Hamiltonian formalism in mechanics which underlies the canonical quantization. It is quite natural to ask whether the above mentioned "polymomentum" Hamiltonian formulations can provide us with a basis for a quantization procedure in field theory. A priori the manifest space-time symmetry of these formulations and the finite dimensionality of the polymomentum phase space can be viewed as potential advantages of such an approach which may look especially appropriate in the context of quantization of General Relativity.

For the first time the problem of field quantization based on a polymomentum formulation was discussed by Born (Born, 1934) and Weyl (Weyl, 1934). I, therefore, refer to the corresponding program as the Born–Weyl quantization. Unfortunately, although the question was not clarified, there were essentially no discussions of the issue since then and

only a few references touching this problem could be cited (see e.g. quotations in (Kanatchikov, 1996)). The main reason for this (besides possible historical ones) seems to be the lack of an appropriate generalization of Poisson brackets to the framework of the polymomentum formulations. However, recently such brackets were constructed within the De Donder–Weyl canonical theory, the simplest representative of the Lepagean canonical theories (see (Kanatchikov, 1996)). Elements of this construction are briefly described in Sect. 2. The purpose of the present communication is to discuss a possible approach to field quantization based on these brackets.

2 DE DONDER–WEYL FORMULATION AND THE POISSON BRACKETS ON FORMS

For the first order Lagrangian field theory given by the Lagrangian density $L = L(y^a, \partial_i y^a, x^i)$, where x^i $(i = 1, ..., n)$ are space-time coordinates and y^a $(a = 1, ..., m)$ are field variables, let us define the *polymomenta* p_a^i and the *De Donder–Weyl (DW) Hamiltonian function* H:

$$p_a^i := \partial L/\partial(\partial_i y^a), \quad H := p_a^i \partial_i y^a - L. \tag{2.1}$$

Then the second order Euler-Lagrange equations can be rewritten in the following first order form (see for instance (Rund, 1966))

$$\partial p_a^i/\partial x^i = -\partial H/\partial y^a, \quad \partial y^a/\partial x^i = \partial H/\partial p_a^i \tag{2.2}$$

which reproduces Hamilton's canonical equations in mechanics when $n = 1$. Therefore eqs. (2.2) can be viewed as a covariant generalization of the Hamiltonian formulation to field theory, to be referred to as the *DW Hamiltonian formulation* in the following. An interesting question is how other elements of the standard Hamiltonian formalism in mechanics can be extended to the present polymomentum formulation of field theory.

An essential ingredient of the canonical formalism is the Hamilton-

Jacobi (HJ) theory which also has its counterpart within the DW formulation. The corresponding DW HJ equation is a partial differential equation for n functions $S^i = S^i(x^j, y^a)$

$$\partial_i S^i + H(x^j, y^a, p^i_a = \partial_a S^i) = 0. \tag{2.3}$$

In order to approach a quantization based on the DW formulation we have to construct an analogue of the Poisson brackets, to identify the canonically conjugate variables, and to find the form of the equations of motion of dynamical variables in Poisson bracket formulation. Here a simplified sketch of the author's recent approach to these questions in the case of scalar field theories is given (see (Kanatchikov, 1996) for more details). Unfortunately, no simple formula for the Poisson bracket is available so far, so that I have to present the whole construction which is a certain generalization of the well-known construction of the Poisson bracket from the symplectic form in mechanics.

Our starting point is what I call the *polysymplectic form* and denote Ω. It generalizes to field theory the symplectic two-form known in mechanics and reduces to the latter at $n = 1$. In local coordinates

$$\Omega := -dy^a \wedge dp^i_a \wedge \omega_i, \tag{2.4}$$

where $\omega := dx^1 \wedge ... \wedge dx^n$ and $\omega_i := \partial_i \rfloor \omega$. Below the variables $z^v := (y^a, p^i_a)$ are called *vertical* and the variables x^i *horizontal*. The polysymplectic form maps functions of the polymomentum phase space variables to vertical multivectors of degree n and, more generally, horizontal q-forms, $\overset{q}{F} := \frac{1}{q!} F_{i_1 \ldots i_q}(z, x) dx^{i_1} \wedge ... \wedge dx^{i_q}$, which play the role of dynamical variables to vertical multivectors of degree $(n-q)$, $\overset{n-q}{X} := \frac{1}{(n-q)!} X^{v i_1 \ldots i_{n-q-1}} \partial_v \wedge \partial_{i_1} \wedge ... \wedge \partial_{i_{n-q-1}}$. Thus for all $0 \leq q < n$

$$\overset{n-q}{X}_{\overset{q}{F}} \rfloor \ \Omega = d^V \overset{q}{F}, \tag{2.5}$$

where $d^V \overset{q}{F} := \frac{1}{q!} \partial_v F_{i_1 \ldots i_q}(z) dz^v \wedge dx^{i_1} \wedge ... \wedge dx^{i_q}$. For $q = n$ a similar map

exists between n-forms $F\omega$ and vertical-vector-valued one-forms $\tilde{X} := \tilde{X}^v{}_k dx^k \otimes \partial_v$: $\tilde{X}_{F\omega} \rfloor \Omega := \tilde{X}^v{}_k dx^k \wedge \partial_v \rfloor \Omega = d^V(F\omega)$.

The Poisson bracket of two forms of degree r and s for which the map (2.5) exists (those are called Hamiltonian) is the following (Hamiltonian) $(r+s-n+1)$-form

$$\{[\overset{r}{F}_1, \overset{s}{F}_2]\} := (-1)^{(n-r)} X_1 \rfloor\ d^V \overset{s}{F}_2 = (-1)^{(n-r)} X_1 \rfloor\ X_2 \rfloor\ \Omega\ . \qquad (2.6)$$

This bracket obeys the axioms of a graded Lie algebra. In particular

$$\{[\overset{r}{F}_1, \overset{s}{F}_2]\} = -(-)^{(n-r-1)(n-s-1)} \{[\overset{s}{F}_2, \overset{r}{F}_1]\}.$$

Moreover, it fulfills a generalized (graded and higher-order) Poisson property (see (Kanatchikov, 1996)). Thus a generalization of Lie and Poisson properties of the standard Poisson bracket, which are known to underlie the standard canonical quantization, is obtained here. The question naturally arises as to whether this new algebraic structure on differential forms can be used as a starting point for a quantization in field theory. Before addressing this question in the next section let us formulate the field equations of motion in terms of the bracket above. One can expect those are given by the bracket with H or $H\omega$. In fact, introducing the total differential $\boldsymbol{d}$ of a form: $\boldsymbol{d}\overset{p}{F} := \partial_i z^v dx^i \wedge \partial_v \overset{p}{F} + dx^i \wedge \partial_i \overset{p}{F}$, which generalizes the total time derivative in mechanics, the equation of motion of a p-form dynamical variable assumes the form

$$\boldsymbol{d}\overset{p}{F} = \{[H\omega, \overset{p}{F}]\} + d^{hor}\overset{p}{F}, \qquad (2.7)$$

where $\{[H\omega, \overset{p}{F}]\} := \tilde{X}_{H\omega} \rfloor d^V \overset{p}{F}$ and $d^{hor}\overset{p}{F} := dx^i \wedge \partial_i \overset{p}{F}$. To prove (2.7) substitute for the components of $\tilde{X}_{H\omega}$ their values on extremals: $\tilde{X}^v{}_k = dz^v/dx^k$. The field equations in the DW canonical form (2.2) are reproduced if, for instance, the forms $p^i_a \omega_i$ and y^a are inserted into (2.7).

3 QUANTIZATION AND GENERALIZED SCHRÖDINGER EQUATION

In order to develop an approach similar to the canonical quantization in the Schrödinger representation of quantum mechanics the pair(s) of the canonically conjugate variables have to be identified. Let us introduce $(n-1)$-forms associated with polymomenta: $p_a := p_a^i \omega_i$. Then the following set of the canonical brackets in Lie subalgebra of 0- and $(n-1)$-forms can be obtained from (2.5) and (2.6)

$$\{[p_a, y^b]\} = \delta_a^b, \quad \{[p_a^i, y^b \omega_j]\} = \delta_j^i \delta_a^b, \quad \{[p_a, y^b \omega_j]\} = \delta_a^b \omega_j. \tag{3.1}$$

We quantize them using Dirac's correspondence rule that the graded Poisson bracket multiplied by $i\hbar$ goes over into the graded commutator with the same symmetry property. In the "y-representation" the quantization of the first bracket in (3.1) yields

$$\hat{p}_a = i\hbar \partial_a. \tag{3.2}$$

Assuming $\hat{p}_a^i = i\hbar \partial_a \otimes \hat{p}^i$ and quantizing (3.1b) we obtain

$$\begin{aligned} [\hat{p}_a^i, \widehat{y^b \omega_j}] &= i\hbar \partial_a \otimes \hat{p}^i \circ \widehat{y^b \omega_j} - (-)^{(n-0-1)(n-(n-1)-1)} \widehat{y^b \omega_j} \circ i\hbar \partial_a \otimes \hat{p}^i \\ &= i\hbar \delta_a^b \hat{p}^i \circ \widehat{\omega_j} + \hat{p}^i \circ \widehat{\omega_j} \hat{y}^b i\hbar \partial_a - \widehat{\omega_j} \circ \hat{p}^i \hat{y}^b i\hbar \partial_a = i\hbar \delta_a^b \delta_j^i, \end{aligned}$$

whence

$$\hat{p}^i \circ \hat{\omega}_j = \delta_j^i, \quad \hat{p}^i \circ \hat{\omega}_j - \hat{\omega}_j \circ \hat{p}^i = 0, \tag{3.3}$$

where $\circ$ denotes the composition of operators. To find a realization of (3.3) let us note that graded symmetry properties of the exterior product and of our Poisson bracket should be incorporated in the algebraic system chosen for the representation. A natural, if not unique, choice seems to be the hypercomplex algebra of the space-time manifold (see e.g. Hestenes, 1966). Firstly, it reduces to the complex algebra in the case of quantum mechanics ($n = 1$); secondly, it unifies the properties of operators $dx\wedge$

and $\partial\rfloor$. On this basis we arrive to the realization of (3.3) in terms of the hypercomplex imaginary units γ_i such that $\gamma_i\gamma_j + \gamma_j\gamma_i = g_{ij}$ (g_{ij} is the space-time metric tensor). One can take For instance, one can take

$$\hat{p}^i = \kappa\gamma^i\gamma, \quad \widehat{\omega}_j = \kappa^{-1}\gamma\gamma_j, \tag{3.4}$$

where $\gamma := i^{\frac{1}{2}n(n-1)}\sigma^{\frac{1}{2}}\gamma_1\gamma_2...\gamma_n$, $\sigma = \mathrm{sign}(\det(g_{ij}))$, so that $\gamma^2 = 1$. The quantity κ of the dimension $[length^{-(n-1)}]$ appears in order to account for the physical dimensions of p^i and ω_i. The absolute value of its inverse is expected to be (formally) infinitesimal as ω_i is essentially an infinitesimal volume element.

The hypercomplex algebra of the space-time manifold appears here as a generalization of the complex algebra in the formalism of quantum mechanics. Therefore the wave function also can be taken to be a hypercomplex-valued function on the configuration space of variables (x^i, y^a), i.e. $\Psi = \psi I + \psi_i\gamma^i + \psi_{ij}\gamma^{[i}\gamma^{j]} + ...$ An analogue of the Schrödinger equation for Ψ can be obtained (guessed) from the requirements that (i) the DW HJ equation (2.3) would appear in the classical limit and (ii) the familiar quantum mechanical Schrödinger equation would be reproduced at $n = 1$. Besides, the observation in Sect. 2 that "the DW Hamiltonian governs the exterior differential" is essential. With these considerations in mind the following generalized Schrödinger equation can be put forward

$$i\hbar\kappa\gamma^i\partial_i\Psi = \widehat{H}\Psi, \tag{3.5}$$

where the quantity κ appears again on dimensional grounds. The left hand side is chosen to be the Dirac operator as it can be viewed at once as a multidimensional generalization of the partial time derivative and, in a sense, as an analogue of the exterior differentiation acting on hypercomplex functions. We show below that this equation does indeed reduce to the DW HJ equation in the quasiclassical limit, at least in the case of scalar fields (see eq. (3.10)).

Let us consider the interacting scalar field theory given by the Lagrangian density $L = \frac{1}{2}\partial_i y^a \partial^i y_a - V(y^a)$. For this system $p_i^a = \partial_i y^a$ and the DW Hamiltonian function $H = \frac{1}{2} p_a^i p_i^a + V(y)$. In order to construct the operator $\widehat{H}$ a realization of $\widehat{p_a^i p_i^a}$ has to be found. From the quantization of the bracket $\{[\frac{1}{2}p_a^i p_i^a, y^b \omega_j]\} = p_j^b$ we find $\widehat{p_a^i p_i^a} = -\hbar^2\kappa^2\triangle$, where $\triangle := \partial_a\partial^a$. Thus

$$\widehat{H} = -1/2\,\hbar^2\kappa^2\,\triangle + V(y). \tag{3.6}$$

To close the system of equations (3.5) it is sufficient to take

$$\Psi = \psi\, I + \psi^i \gamma_i.$$

Then (3.5) reduces to the system of equations

$$i\hbar\kappa\partial_i\psi^i = \widehat{H}\psi, \quad i\hbar\kappa\partial_i\psi = \widehat{H}\psi_i \tag{3.7}$$

which gives rise to the conservation law

$$\partial_i[\bar{\psi}\psi^i + \psi\bar{\psi}^i] = \frac{i\hbar\kappa}{2}\partial^a[\bar{\psi} \overleftrightarrow{\partial}_a \psi + \bar{\psi}^i \overleftrightarrow{\partial}_a \psi_i]. \tag{3.8}$$

This could lead to a prescription for the calculation of the expectation values. However, the corresponding scalar product is unlikely to stay positive definite. Moreover, the obstacles of pure algebraic nature are known to a generalization of the quantum theoretical formalism to amplitudes different from the real, quaternion or octonion valued functions (see e.g. (Adler, 1992)). A possible way out can be in replacing the hypercomplex wave function in (3.5) with a spinor one, while still keeping representing operators corresponding to forms in terms of the hypercomplex numbers.

In order to consider the quasiclassical limit of (3.5) let us take the following hypercomplex generalization of the quasiclassical Ansatz for the wave function

$$\Psi = R\,\exp(iS^i\gamma_i/\hbar\kappa), \tag{3.9}$$

where the exponential function is defined via the series expansion. Substituting (3.9) into (3.5) and performing a slightly tedious calculation the

result can be represented in the form

$$\partial_i S^i = -\frac{1}{2}\partial_a S^i \partial_a S_i - V(y) + \frac{1}{2}\hbar^2\kappa^2 \frac{\triangle R}{R} \tag{3.10}$$

which reproduces the DW HJ equation for scalar fields (cf. eq. (2.3)) in the quasiclassical limit $\hbar\kappa \to 0$. However, besides the DWHJ equation two other conditions on the "HJ functions" S^i: $\partial_i S^i = \frac{S^i}{|S|}\partial_i |S|$ and $\partial_a S^i \partial^a S_i = \partial_i S^i$, where $|S| := \sqrt{S_i S^i}$, arise here, so that no *complete* DWHJ theory is reproduced. It is interesting to note that the "quantum potential" term in (3.10), $\hbar^2\kappa^2 \triangle R/R$, is of the similar form as in quantum mechanics (cf. Bohm e.a., 1987).

For a free real scalar field $\widehat{H} = -\frac{1}{2}\hbar^2\kappa^2\partial_{yy} + \frac{1}{2}\frac{m^2}{\hbar^2}y^2$, and eq. (3.5) can be solved by the separation of variables: $\Psi(x,y) = \Phi(x)f(y)$, where $\Phi(x) := \phi(x) + \phi_i(x)\gamma^i$ and $f(y)$ is a function. This leads to the eigenvalue problem for the DW Hamiltonian operator: $\widehat{H}f = \chi f$. In the present case it is just the harmonic oscillator problem in the space of field variables, so that the eigenvalues of $\widehat{H}$ are $\chi_N = \kappa m(N + 1/2) =: \kappa m_N$ and the eigenfunctions are those of the harmonic oscillator. The scalar part of Φ satisfies the Klein-Gordon equation $\Box\phi = -\chi^2/\hbar^2\kappa^2\phi$, and the vector part obeys $\phi_i = \frac{i\hbar\kappa}{\chi}\partial_i\phi$. Let us note that the quantity κ cancels out in the equations governing the space-time behavior of Ψ. Now, for the ground state, $N = 0$, the scalar part of Ψ assumes the form

$$\psi_{0,\mathbf{k}}(y, \mathbf{x}, t) \sim e^{i\omega_{0,\mathbf{k}}t - i\mathbf{k}\cdot\mathbf{x}} e^{-\frac{m}{2\kappa\hbar^2}y^2}, \tag{3.11}$$

where $\omega_{0,\mathbf{k}}^2 - \mathbf{k}^2 = (\frac{1}{2}\frac{m}{\hbar})^2$ and a normalization factor is omitted. In general, any solution of eqs. (3.7) is a linear combination of $\Psi_{N,\mathbf{k}}$.

Ignoring the problem with the positive indefiniteness of the scalar product implied by (3.8) our hypercomplex wave function $\Psi(y, x)$ can be interpreted as the probability amplitude of finding the value y of the field in the infinitesimal vicinity of the space-time point x. If so, we can try to relate it to the Schrödinger wave functional in quantum field theory

$\Psi([y(\mathbf{x})],t)$ which is the probability amplitude of finding a field configuration $y(\mathbf{x})$ on a space-like hypersurface of constant time t.

For instance, for a free scalar field the Schrödinger vacuum state functional

$$\Psi_0([y(\mathbf{x})],t) = \eta\ \exp\left(\frac{i}{\hbar}E_0 t - \frac{1}{\hbar}\int d\mathbf{x}\, y(\mathbf{x})\frac{1}{2}(-\nabla_{\mathbf{x}}^2 + m^2/\hbar^2)^{\frac{1}{2}} y(\mathbf{x})\right) \tag{3.12}$$

can be expressed as the infinite product of the harmonic oscillator wave functions over all points in the $\mathbf{k}$-space (cf. e.g. Hatfield, 1992)

$$\lim_{V\to\infty} \eta\ \prod_{\mathbf{k}} \exp\frac{1}{2}\left(i\omega_{\mathbf{k}} t - \frac{1}{V\hbar}\ \omega_{\mathbf{k}} \tilde{y}^2(\mathbf{k})\right), \tag{3.13}$$

where $\omega_{\mathbf{k}} := (m^2/\hbar^2 + \mathbf{k}^2)^{\frac{1}{2}}$, $E_0 = \frac{1}{2}\hbar \int_V d\mathbf{x} \int \frac{d\mathbf{k}}{(2\pi)^{n-1}}\omega_{\mathbf{k}}$ is the (divergent) vacuum state energy, V is an "infinitely large" volume element, η is a normalization factor, and the Fourier series expansion $y(\mathbf{x}) = \frac{1}{V}\sum_{\mathbf{k}} \tilde{y}(\mathbf{k})e^{i\mathbf{kx}}$ is used in passing from (3.12) to (3.13).

At the same time the amplitude of finding the configuration $y(\mathbf{x})$ of the field can be composed from the infinite set of the amplitudes of finding the corresponding values $y = y_{\mathbf{x}}$ of the field in the points $\mathbf{x}$ of the equal-time hypersurface. These amplitudes are given by our wave function $\Psi(y = y_{\mathbf{x}}, x = (\mathbf{x}, t))$. Now, if the correlations between the values of the field in space-like separated points are neglected, the composed vacuum state amplitude can be written as the infinite product over all points of the $\mathbf{x}$-space of the lowest eigenvalue solutions (3.11), that is

$$\prod_{\mathbf{k}} e^{i\omega_{0,\mathbf{k}} t - i\mathbf{k}\cdot\mathbf{x}} \prod_{\mathbf{x}\in V} \exp\left(-\frac{m}{2\kappa\hbar^2} y_{\mathbf{x}}^2\right),$$

where the product over $\mathbf{k}$ accounts for filling of all $\mathbf{k}$-states in the vacuum state. Inserting the formal Fourier series expansion for $y_{\mathbf{x}}$ and using a discretization in both $\mathbf{k}$- and $\mathbf{x}$- space and the identity $\prod_{\mathbf{k}} e^{i\mathbf{kx}} = 1$ this expression can be transformed to the form similar to (3.13)

$$\lim_{\substack{V\to\infty \\ Q\to\infty}} \prod_{\mathbf{k}} \exp\left(i\omega_{0,\mathbf{k}} t - \frac{Q}{2(2\pi)^{n-1}V\hbar^2\kappa}\ m\ \tilde{y}^2(\mathbf{k})\right), \tag{3.14}$$

where $Q := \int_Q d^{n-1}\mathbf{k}$ plays the role of the ultra-violet cutoff.

The amplitude in (3.14) is different from that in (3.13) in two respects. Firstly, in the second term of (3.14) we have obtained the proper mass m instead of the frequency $\omega_{\mathbf{k}}$ in (3.13). This is probably a result of our above neglect of space-like correlations which appear in the standard theory due to the non-local character of the operator $\sqrt{m^2/\hbar^2 - \nabla_{\mathbf{x}}^2}$ in (3.12), and which are accounted for in the part of the Feynman propagator non-vanishing at space-like separations. However, the study of Green functions of the second-order consequence of (3.7): $\hbar^2\Box\psi = -\frac{1}{\kappa^2}\widehat{H}^2\psi$, demonstrates that these correlations are not beyond the scope of the present approach, so that the neglect of space-like correlations is of a technical character. Secondly, we face the problem of a proper interpretation of the parameter κ. The latter appeared in (3.4) as essentially the inverse of an infinitesimal $(n-1)$-volume element. It can, therefore, naturally be identified with the ultra-violet cutoff scale Q divided by $(2\pi)^{n-1}$. With such an identification eq. (3.14) provides us with a very long wave ($|\mathbf{k}| \ll m$) limit (in which the space-like correlations are vanishing) of the Schrödinger vacuum state functional (3.13). Therefore, the composed amplitude in (3.14) appears to be consistent with the standard result in (3.12) within the simplifying rough approximation of neglect of non-vanishing part of the Feynman propagator at space-like separations.

Summarizing, we have argued that a quantization of field theory based on the polymomentum Hamiltonian formulation of De Donder-Weyl leads to an interesting hypercomplex generalization of the formalism of quantum theory which is different from the previously considered versions of the quaternionic quantum mechanics and related approaches (cf. Adler, 1995). However, serious efforts are still required for understanding of how the present approach to quantization can be related to or complement the modern notion of the quantum field.

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